Section 1-6, Mathematics 108

Complex Numbers

looking at the equation $x^2+1=0$ we see that there are no solutions in the real numbers.

We extend the reals by defining a value $i=\sqrt{-1}$ which gives us

the **imaginary** numbers
$\left\{x: x=ai \textit{ where } a\in\mathbb{R} \textit{ and } i=\sqrt{-1}\right\}$

We then define the **complex** numbers as

$\left\{a+bi: a,b\in\mathbb{R}\right\}$

Adding Complex numbers

Example:

$(5+2i)+(6+3i)=(5+6)+(2+3)i=11+5i$

Multiplying Complex numbers

Here we can use **FOIL**

$(5+2i)(6+3i)=(5\cdot 6)+(5\cdot 3i)+(2i\cdot 6)+(2i\cdot 3i)=$
$30+15i+12i+6i^2$

But since $i=\sqrt{-1}$ we have $i^2=-1$ so we get

$30+15i+12i+6i^2=30+27i-6=24+27i$

Complex Conjugates

We note that that $(A+Bi)(A-Bi) = A^2 - B^2i^2 = A^2 + B^2$ is always a real number.

We define the **complex conjugate of** *a*+*bi* as *a*-*bi*.

One use of the complex conjugate is that when dividing two complex numbers we can make the denominator of our fraction a real.

Example:

$$(5+3i)\div(2+i) = \frac{5+3i}{2+i} = \frac{5+3i}{2+i}\cdot\frac{2-i}{2-i} = \frac{10+6i-3i+3}{4+1} = \frac{13+3i}{5} = \frac{13}{5}+\frac{3}{5}i$$

A note about *i*

$i^2 = -1$ but also $(-i)^2 = (-1)^2 i^2 = -1$

So the equation $x^2 + 4 = 0$ has two solutions, 2*i* and -2*i*.

Quadratic Equations over the Reals

Recall that solution of $ax^2 + bx + c = 0$ has no solutions in the reals if the discriminant $b^2 - 4ac < 0$

However over the complex numbers there are 2 solutions.

Example:

$$x^2 + 4x + 5 = 0$$

$$x = \frac{-4 \pm \sqrt{16-20}}{2} = \frac{-4 \pm \sqrt{-4}}{2} = \frac{-4 \pm 2i}{2} - 2 \pm i$$

Note that the two solutions will always be complex conjugates.